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TRANSVERSE VIBRATIONS OF ISOTROPIC, THIN CIRCULAR PLATES WITH A RECTANGULARLY ORTHOTROPIC CIRCULAR CORE

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1. INTRODUCTION

The present note is an outgrowth of previous publications [1–4] and deals with the determination of the fundamental frequency of transverse vibration of the system as shown in Figure 1.

For the sake of generality, it is assumed that a point mass M is rigidly attached to the plate center.

The optimized Rayleigh–Ritz method is employed to determine the fundamental eigenvalue and an independent solution is obtained using the finite element procedure [5]. Good engineering agreement is observed between both sets of results.

2. APPROXIMATE SOLUTION

In the case of normal modes of vibration the dynamic behavior of the system is governed by the functional:

$$J(W) = D\iint_{\bar{P}_1} [(W_{\bar{x}^2} + W_{\bar{y}^2})^2 - 2(1-\upsilon)(W_{\bar{x}^2}W_{\bar{y}^2} - W_{\bar{x}\bar{y}}^2)]\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y}$$

$$+ \iint_{\bar{P}_2} (D_1 W_{\bar{x}^2}^2 + 2D_1\upsilon_2 W_{\bar{x}^2}W_{\bar{y}^2} + D_2 W_{\bar{y}^2}^2 + 4D_k W_{\bar{x}\bar{y}}^2)\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y}$$

$$- \rho h\omega^2 \iint_{\bar{P}} W^2\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} - M\omega^2 W^2(0,0) \qquad (1)$$

and appropriate boundary conditions.

Substituting: $D_1' = D_1/D$, $D_2' = D_2/D$, $D_k' = D_k/D$ and $\bar{x} = ax$, $\bar{y} = ay$ into equation (1) one obtains

$$\frac{a^2}{D}J(W) = \iint_{P_1} [(W_{x^2} + W_{y^2})^2 - 2(1-\upsilon)(W_{x^2}W_{y^2} - W_{xy}^2)]\,\mathrm{d}x\,\mathrm{d}y$$

$$+ \iint_{P_2} (D_1' W_{x^2}^2 + 2D_1'\upsilon_2 W_{x^2}W_{y^2} + D_2' W_{y^2}^2 + 4D_k' W_{xy}^2)\,\mathrm{d}x\,\mathrm{d}y$$

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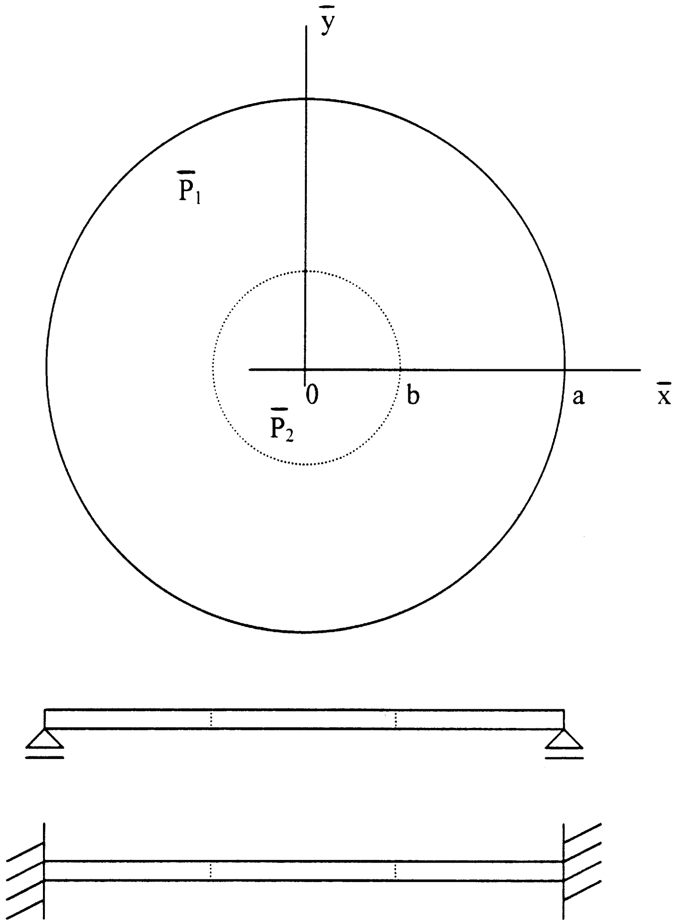

Figure 1. Vibrating mechanical system under study (note: $\bar{P}_1$ = isotropic portion, $\bar{P}_2$ = orthotropic core; $\bar{P} = \bar{P}_1 \cup \bar{P}_2$).

$$- \Omega^2 \left[\iint_P W^2 \,\mathrm{d}x \,\mathrm{d}y + \pi\mu W^2(0,0) \right], \tag{2}$$

where $\Omega^2 = (\rho h a^4/D)\omega^2$, $\mu = M/M_p$; M_p is the plate mass.

In order to apply the Rayleigh–Ritz method one makes

$$W(x, y) \cong W_a = \sum_{j=1}^{N} C_j \varphi_j(x, y) \tag{3}$$

and applying Ritz minimization condition one obtains

$$\frac{a^2}{2D}\frac{\partial J}{\partial C_i} = \sum_{j=1}^{N} \left\{ \iint_{P_1} (\varphi_{jx^2} + \varphi_{jy^2})(\varphi_{ix^2} + \varphi_{iy^2}) \right.$$

$$- (1 - \upsilon)(\varphi_{jy^2}\varphi_{ix^2} + \varphi_{jx^2}\varphi_{iy^2} - 2\varphi_{jxy}\varphi_{ixy})$$

TABLE 1

Fundamental frequency coefficients $\Omega_1 = \sqrt{\rho h/D}\,\omega_1 a^2$†

	μ	(A) $r_b = 0$	(B) 0·2	(C) 0·4	(D) 0·6	(E) 0·8
	0	4·93(1)	4·52	4·27	4·13	4·03
		(4·94)(2)	(4·83)	(4·57)	(4·31)	(4·11)
SS	0·2	3·76	3·42	3·24	3·14	3·08
		(3·76)	(3·62)	(3·42)	(3·24)	(3·11)
	0·4	3·15	2·86	2·70	2·63	2·58
		(3·13)	(3·00)	(2·83)	(2·69)	(2·58)
	0	10·20	9·58	9·39	9·20	8·79
		(10·22)	(9·93)	(9·60)	(9·37)	(9·02)
C	0·2	7·00	6·51	6·38	6·21	5·92
		(6·87)	(6·46)	(6·28)	(6·12)	(5·89)
	0·4	5·64	5·23	5·13	4·98	4·75
		(5·47)	(5·10)	(4·95)	(4·82)	(4·65)

Note: One quarter of the plate domain was subdivided into: (A) 5069 elements, (B) 5066, (C) 5076, (D) 5081, (E) 5094 respectively.

†Comparison of analytical and finite element values: (1) Optimized Rayleigh–Ritz method. (2) Finite element predictions.

$$+ \iint_{P_2} [D'_1\, \varphi_{jx^2}\, \varphi_{ix^2} + D'_1\, \upsilon_2 (\varphi_{jy^2}\, \varphi_{ix^2} + \varphi_{jx^2}\, \varphi_{iy^2})$$

$$+ D'_2\, \varphi_{jy^2}\, \varphi_{iy^2} + 4D'_k\, \varphi_{jxy}\, \varphi_{ixy}]\, \mathrm{d}x\, \mathrm{d}y$$

$$- \Omega^2 \left[\iint_P \varphi_j\, \varphi_i\, \mathrm{d}x\, \mathrm{d}y + \pi\mu\varphi_j(0)\, \varphi_i(0) \right] \Bigg\} C_j = 0. \qquad (4)$$

In the present study, $N = 2$ and

$$\varphi_1 = \alpha_1 r^p + \beta_1 r^2 + 1, \qquad \varphi_2 = \alpha_2 r^{p+2} + \beta_2 r^4 + r^2, \qquad r = \sqrt{x^2 + y^2}, \qquad (5)$$

where the α's and β's are obtained substituting each co-ordinate function in the boundary conditions for the cases of simply supported and clamped edges respectively.

Once the fundamental eigenvalue is determined one minimizes it with respect to "p" (see equations (5)).

3. NUMERICAL RESULTS

Calculations of fundamental frequency coefficients were performed for the following constitutive characteristics and geometric parameters: $\upsilon_2 = \upsilon = 0{\cdot}30$, $D_1/D = 4/5$, $D_2/D_1 = 1/2$, $D_k/D_1 = 1/3$; $r_b = b/a = 0$, 0·2, 0·4, 0·6, 0·8; $\mu = 0$, 0·2, 0·4.

Table 1 depicts values of $\Omega_1 = (\sqrt{\rho h/D})\,\omega_1\, a^2$ obtained by means of the optimized Rayleigh–Ritz method and the finite element technique for simply supported and clamped plates. The agreement is good from an engineering viewpoint. Some analytical results are lower than the finite element values. However, since the latter are presumably very accurate in view of the large number of elements used, this fact may be due to a slight numerical instability when making use of the optimized variational approach.

The model presented is also a first order approximation for the situation where an isotropic circular plate has been damaged in a central portion and acquires orthotropic characteristics.

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